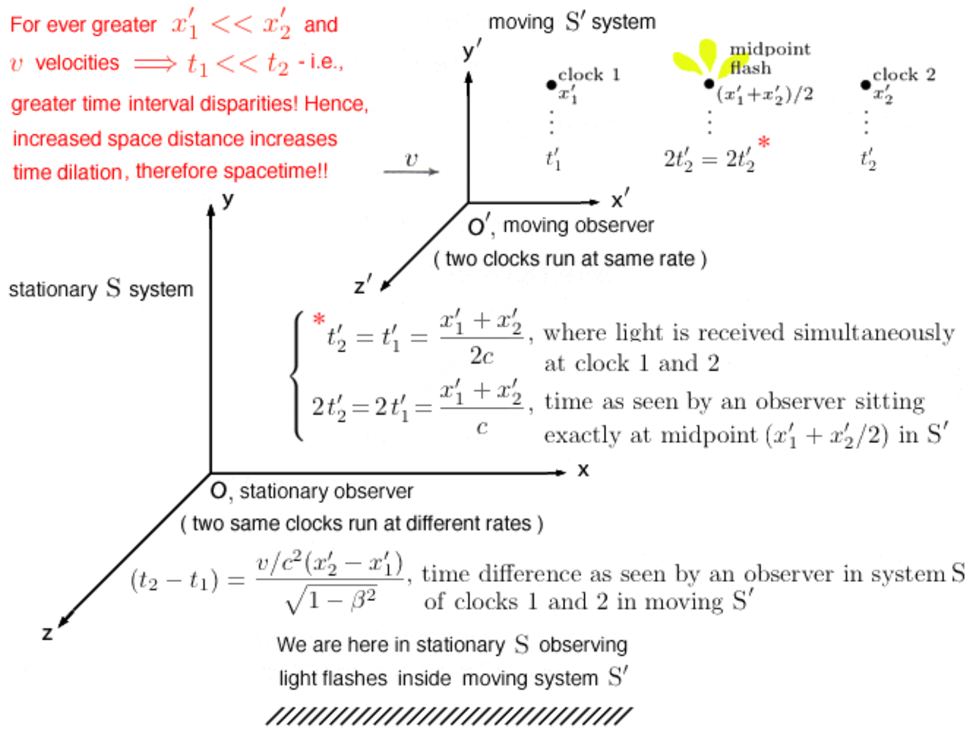
Some Results of the Lorentz Transformation Equations



$$t = \frac{t' + (v/c^2)\,x'}{\sqrt{1-\beta^2}}, \text{ where } \beta^2 = \left(\frac{v}{c}\right)^2$$

$$\Longrightarrow (t_2 - t_1)_{|\text{S-time interval}} = \frac{t'_2 + \frac{v}{c^2}x'_2 - t'_1 - \frac{v}{c^2}x'_1}{\sqrt{1-\beta^2}}$$

$$\therefore \quad (t_2 - t_1)_{|\text{S-time interval}} = \frac{(t'_2 - t'_1) + \frac{v}{c^2}(x'_2 - x'_1)}{\sqrt{1-\beta^2}}$$

I. Result 1 - clock rates:

From the above equation for "S - time interval" (that is, time as observed in system S), it is obvious that the *units of time* for clock 2 will be greater as compared to *units of time* for clock 1 in moving system S'. Why?

> Because t_1 and t_2 are each a function of distances respectively x'_1 and x'_2, therefore the greater will be $t_1 << t_2$ for greater $x'_1 << x'_2$, and hence the *units of time* for each t-time will be adjusted accordingly.

Furthermore, since whenever

$$x'_1 << x'_2$$

clock 2 is much farther away from clock 1 in moving frame system S' - the slower clock will be the clock at x'_2 than at x'_1 to an outside stationary observer in frame S, and hence an

$$(t_2 - t_1)_{|\text{S-time interval}}$$

will also be greater signifying a slower rate of time passing in frame system S' as viewed by an observer in frame system S !!

Ia. Corollary - space-time:

The greater the distance separating clocks 1 and 2, the slower will be the rate of time passing to an outside stationary observer!

This phenomenon has already been demonstrated as for when the further distant clock 2 at x'_2 runs slower than the nearer clock 1 at x'_1 in relatively moving frame system S' to an outside observer in stationary system S.

The conclusion is therefore inescapable: *time is dependent on space* as these Special Relativity equations demonstrate!!

II. Result 2 - "The Failure of Simultaneity of Time at great distances":

Whenever

$$2t'_2 = 2t'_1\text{, where light is received simultaneously at clock 1 and 2 as seen by an observer sitting exactly at midpoint } (x'_1 + x'_2/2) \text{ in } \mathrm{S}'\text{, then}$$

$$\Longrightarrow \; t'_2 = t'_1\text{, definition of time simultaneity}$$

$$\Longrightarrow \; (t_2 - t_1) = \frac{v/c^2(x'_2 - x'_1)}{\sqrt{1-\beta^2}}\text{, time difference as seen by an observer in system S of clocks 1 and 2 moving in } \mathrm{S}'.$$

And for ever greater separating distances for clocks 1 and 2,

$$x'_1 << x'_2\,,$$

there will be ever greater disparities in time of light received respectively at clocks 1 and 2 for a stationary observer in frame system S as shown by

$$t_1 << t_2$$

In other words, in physical reality there is "*no simultaneity of clock events*" when either great distances or great velocities of clocks are involved relative to a stationary observer!!

III. Result 3 - Length Contraction:

Again, as between stationary system S and frame system S' moving away at relative velocity v, we have

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and for a rigid rod fixed at

$$(x'_1, 0, 0) \text{ and } (x'_2, 0, 0)$$

in the "moving away" frame system S', we have length

$$l' = (x'_2 - x'_1) \neq 0\,.$$

Now for a moving observer in S' at some arbitrary time, t, we therefore have

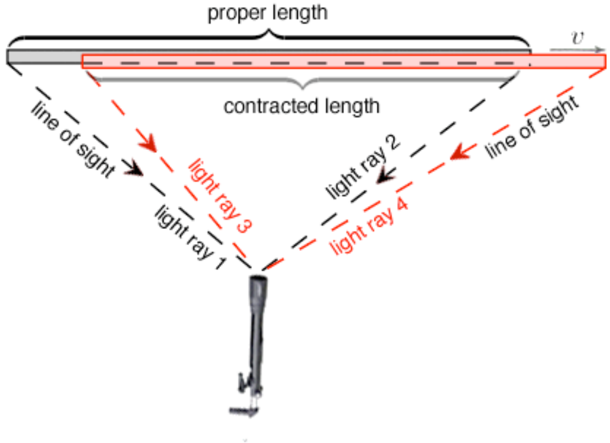
$$\Longrightarrow \quad \begin{aligned} l' = (x'_2 - x'_1) &= \frac{(x_2 - vt) - (x_2 - vt)}{\sqrt{1-\beta^2}} = \frac{(x_2 - x_1)}{\sqrt{1-\beta^2}} \\ (x_2 - x_1) &= l'\sqrt{1-\beta^2} \\ \therefore \; l_{(S\,\text{system})} &= l'\sqrt{1-\beta^2}\,. \end{aligned}$$

Therefore in stationary frame systems, S , the rigid rod will appear to shrink in the longitudinal $x\,(x')$ - axis direction by the inverse of the *Lorentz Factor*

$$\frac{1}{\gamma} = \sqrt{1-\beta^2}\text{, where } \beta^2 = \frac{v^2}{c^2} \text{ and } \gamma \text{ is called the } \textit{Lorentz Factor}.$$

That is, for an observer in S, a rigid rod in "moving away" frame system S' will appear to shrink by an amount given by the *Lorentz Factor*, and equally for a relatively "moving away" system S for a stationary observer in system S', this same rod will also appear to be contracted!! It's all relative! And it's called reciprocal length contraction.

This contraction effect is called the *Lorentz Contraction Effect*.



note: Proper length = (light distance ray 1 - light distance ray 2)

Relativistic length = (light distance ray 3 - light distance ray 2) since light ray 3 arrives sooner than the more distant light ray 4 due to the **universal constancy** of the speed of light

Conclusion: apparent length contraction!

Reality? The practical, operational reality is what is being perceived from great distance and at significant magnitudes to the upper limit of the speed of light.

However in general relativity due to the non - inertial effects of acceleration, length contraction is a measurable reality.

And in order, therefore, to maintain a universal constant speed of light in any light sphere in any direction by Einstein's special relativity proposition, longitudinal length contraction must be invoked!

More simply, ***length contraction is imputed*** in order to maintain a universal constant speed of light when determining time dilation in both Einstein's Special and General Relativity equations!

$$c_{(\text{speed of light})} = \frac{l_{(\text{relativistic length})\downarrow}}{t_{(\text{relativistic time})\uparrow}} = \underline{\text{universal constant}} \text{ throughout the universe}$$

IV. Result 4 - Time Dilation (time interval increase):

In this case, let there be just one clock at, say, x'_1, hence

$$x'_2 = x'_1$$

and assume time

$$t'_2 > t'_1 \text{ as time passes "normally" at } x'_1 \text{ in S}',$$

then

$$(t_2 - t_1) = \frac{t'_2 - t'_1 + v/c^2(x'_2 - x'_1)}{\sqrt{1-\beta^2}} = \frac{(t'_2 - t'_1)}{\sqrt{1-\beta^2}}$$

reduces down to

$$t_{(\text{dilated time})} = \frac{t'}{\sqrt{1-\beta^2}}, \text{which is the ad-hoc Michelson-Morley assumption}$$

for observations of S' being made from S.

Conversely this will also be true for the inverse

$$t'_{(\text{dilated time})} = \frac{t}{\sqrt{1-\beta^2}},$$

where observations of system S are being made from S'. This type of time dilation for non - accelerating, inertial system motion is mutually reciprocal which precludes "The Twin Clock Paradox" construct.

Video: Time Dilation Experiment

source: "Time Dilation - An Experiment with Mu - Mesons", ©1962, presented by The Science Teaching Center of the Massachusetts Institute of Technology with the support of the National Science Foundation, demonstrated by Profs. David H. Frisch, M.I.T. and James H. Smith, University of Illinois. Video source: https://www.youtube.com/watch?v=2e9ltbbOwtc

Lorentz Transformation Rules Summary

Rule 1: Every clock will appear to go at its fastest rate when it is at rest relative to the observer; hence, any motion relative to an observer slows the apparent rate of any clock.

Rule 2: Every rigid rod will appear to be at its greatest longitudinal extent when it is at rest relative to the observer, whereas transverse or perpendicular extants relative to the direction of motion are always uneffected. Therefore any longitudinal motion relative to an observer shrinks any rigid rod in the direction of motion by an amount given by the Lorentz factor.

Lorentz Inverse Transformation Equations Example*

* note : this example is used in some future upcoming Relativity Science Calculator Mac application

An unknown particle, γ_{strange} , appears and then disappears with a "lifetime" of 1.80 x 10^{-8} sec in a particle accelerator such as CERN's LHC (Large Hadron Collider) and is observed during it's "lifetime" to have a concurrent velocity of 0.99 c together in a beam of (other) known particles.

(i). Determine the proper lifetime for γ_{strange}:

The proper lifetime (proper time) is the lifetime of the particle measured by an observer moving coincident with the particle in the particle's own frame of reference, S' system.

Therefore by relativistic time dilation,

$$t = \frac{t'}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ where } t \,\epsilon\, \mathrm{S} \text{ and } t' \,\epsilon\, \mathrm{S}'$$

$$\Longrightarrow t' = t\sqrt{1 - \frac{v^2}{c^2}}, \ \underline{\text{proper time}}^{*} \text{ in } \mathrm{S}'$$

$$= (1.8 \times 10^{-8}\,\text{sec})\sqrt{1 - (0.99)^2}, \text{ since } v = 0.99c$$

$$= (1.8 \times 10^{-8}\,\text{sec})\sqrt{1 - (0.9801)}$$

$$= (1.8 \times 10^{-8}\,\text{sec})\sqrt{0.0199}$$

$$\Longrightarrow t' = (1.8 \times 10^{-8}\,\text{sec})(0.1410673)$$

$$\Longrightarrow \therefore\ t' = 0.2539212, \times 10^{-8}\,\text{sec}, \ \underline{\text{proper time}}^{*} \text{ in } \mathrm{S}'$$

*note: see The Minkowski Space - Time Light Cone for the following more formal derivation:

$$\therefore \boxed{\tau^2 = \Delta t^2 - \frac{1}{c^2}\Delta d^2}, \underline{\text{proper time.}}$$

$$\therefore \boxed{d\tau = \frac{1}{c}\sqrt{c^2dt^2 - dx_1^2 - dx_2^2 - dx_3^2}}, \underline{\text{proper time interval}}$$

source: "Space and Time", H. Minkowski, Cologne, 1908

(ii). Determine distance travelled for γ_{strange}:

The particle accelerator resides in (actually comprises) stationary system S, therefore we use

$$\left.\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + \left(v/c^2\right)x'}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}\right\} \text{Lorentz Inverse Transformation Equations}$$

where

$$\begin{aligned} \text{S} &= \text{S}(x, y, z, t) \\ \text{S}' &= \text{S}'(x', y', z', t') \end{aligned}$$

Since we are trying to determine distance travelled in our particle accelerator, or stationary S system, we must place ourselves as "observers" exactly within the particle's own frame of reference (that is, relatively moving system S'), so as to recreate the coordinates for the appearance and disappearance of the γ_{strange} particle in CERN's LHC (Large Hadron Collider):

$$\begin{aligned} \text{appearance}: \ (x'_o, y'_o, z'_o, t'_o) &= (0, 0, 0, 0) \\ \text{disappearance}: \ (x'_1, y'_1, z'_1, t'_1) &= (0, 0, 0, t'_1), \end{aligned}$$

since because we are *imagining* as moving coincident (actually imagining as being stationary) with γ_{strange}, therefore we're (relatively) stationary with γ_{strange} and S', hence $x'_1 = 0$.

Going back to our stationary system S, the particle accelerator, from which we are making these "outside" (external) observations, <u>Lorentz Inverse Transformation Equations</u> give

$$\begin{aligned} x_o &= \frac{x'_0 + vt'_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{0 + v(0)}{\sqrt{1 - \frac{v^2}{c^2}}} = 0 \\ y_o &= y'_o = 0 \\ z_o &= z'_o = 0 \\ t_o &= \frac{t'_o + (v/c^2)x'_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{0 + (v/c^2)(0)}{\sqrt{1 - \frac{v^2}{c^2}}} = 0 \end{aligned}$$

whereas

$$\begin{aligned} x_1 &= \frac{x'_1 + vt'_1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{0 + vt'_1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt'_1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y_1 &= y'_1 = 0 \\ z_1 &= z'_1 = 0 \\ t_1 &= \frac{t'_1 + (v/c^2)x'_1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t'_1 + (v/c^2)(0)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t'_1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}$$

Hence relative to stationary S or particle accelerator, our Lorentz Inverse Transformation Equations become

$$(x_1 - x_o) = \frac{vt_1'}{\sqrt{1-\frac{v^2}{c^2}}} - 0 = \frac{vt_1'}{\sqrt{1-\frac{v^2}{c^2}}},\ \gamma_{\text{strange}}\ -\ \text{distance travelled}$$
$$(y_1 - y_o) = 0$$
$$(z_1 - z_o) = 0$$
$$(t_1 - t_o) = \frac{t_1'}{\sqrt{1-\frac{v^2}{c^2}}} - 0 = \frac{t_1'}{\sqrt{1-\frac{v^2}{c^2}}},\ \gamma_{\text{strange}}\ -\ \text{time travelled or "lifetime"}$$

Finally,

$$(x_1 - x_o) = \frac{\overbrace{(0.99c)}^{\substack{\gamma_{\text{strange}}\\ \text{velocity}}}\overbrace{(0.2539212\ \times\ 10^{-8}\text{sec})}^{\gamma_{\text{strange}}\ \text{proper time}}}{\sqrt{1-(0.99)^2}}$$
$$\Longrightarrow (x_1 - x_o) = \frac{(0.99)\overbrace{(2.998\ \times\ 10^8\,\text{m-sec}^{-1})}^{\text{speed of light}}\overbrace{(0.2539212\ \times\ 10^{-8}\text{sec})}^{\gamma_{\text{strange}}\ \text{proper time}}}{\sqrt{1-(0.99)^2}}$$
$$\Longrightarrow \therefore\ (x_1 - x_o) = 5.3424373\,\text{m}\ \approx\ 5.34\,\text{m},\ \gamma_{\text{strange}} - \text{distance travelled}$$

in time

$$\Longrightarrow \therefore\ (t_1 - t_o) = \frac{\overbrace{(0.2539212\ \times\ 10^{-8}\text{sec})}^{\gamma_{\text{strange}}\ \text{proper time}}}{\sqrt{1-(0.99)^2}} = 1.80\ \times\ 10^{-8}\ \text{sec},\ \gamma_{\text{strange}} - \text{"lifetime"}$$

Notice that

$$(t_1 - t_o) = 1.80\ \times\ 10^{-8}\,\text{sec}\,\epsilon\,\mathrm{S} \gg t' = 0.2539212\ \times\ 10^{-8}\,\text{sec}\,\epsilon\,\mathrm{S}'$$

which means that any (atomic) clock "attached" to γ_{strange} will move ***slower*** as seen (i.e., measured) by an observer in stationary system S as compared to an observer of time "attached" to this particle; this is the meaning of time dilation or time interval expansion.

(iii). If time dilation did not exist in nature:

This means that hypothetically the speed of light is infinite or instantaneous, in which case

$$\Delta t = (t_1 - t_o)\,\epsilon\,\mathrm{S}\ = \frac{t_1'}{\sqrt{1-0^2}} = \frac{t_1'}{1} = t_1'\,\epsilon\,\mathrm{S}',\ \text{since}$$
$$c\ \rightarrow\ \infty\,\text{implies}\,v/c = 0$$

and

$$\Delta x = (x_1 - x_o)\,\epsilon\,\mathrm{S} = \frac{vt_1'}{\sqrt{1-0^2}} = \frac{vt_1'}{1} = vt_1',\ \text{since}$$
$$c\ \rightarrow\ \infty\,\text{implies}\,v/c = 0$$

Or, the time viewed in system S will the same as the time viewed in system S′, and hence

$$(x_1 - x_o)\,\epsilon\,\mathrm{S} = \overbrace{(0.99\ \times\ 2.998\ \times\ 10^8\,\text{m-sec}^{-1})}^{\substack{\gamma_{\text{strange}}\\ \text{velocity}}}\cdot\overbrace{(0.2839212\ \times\ 10^{-8}\text{sec})}^{\substack{\gamma_{\text{strange}}\\ \text{proper time}}}$$
$$\Longrightarrow \therefore\ (x_1 - x_o)\,\epsilon\,\mathrm{S} = 0.7536432\,\text{m},\ \gamma_{\text{strange}}\ -\ \text{distance moved in S, where}$$
$$(x_1' - x_o')\,\epsilon\,\mathrm{S}' = 0,\ \text{still zero, since the observer in}\ \mathrm{S}'\ \text{is stationary in}\ \mathrm{S}'.$$

Finally and continuing with the hypothetical that time dilation is not a reality in nature, also assume that γ_{strange} possesses a velocity considerably less than *c*, speed of light, or $v \ll c$, then

$$(x_1 - x_o)\,\epsilon\,\mathrm{S} = 0, \text{ since } v \ll c \text{ implies } v \rightarrow 0 \text{ and } v/c = \frac{0}{\infty} = 0$$

That is,

$$(x_1 - x_o)\,\epsilon\,\mathrm{S} \cong \overbrace{(0.00 \times 2.998 \times 10^8\,\text{m-sec}^{-1})}^{\text{velocity}} \cdot \overbrace{(0.2839212 \times 10^{-8}\text{sec})}^{\text{proper time}}, \text{ as } v \rightarrow 0$$
$$\Longrightarrow \therefore\ (x_1 - x_o)\,\epsilon\,\mathrm{S} = 0.0\,\mathrm{m}$$