de Broglie's Quantum Equations

"When you can measure what you are speaking about and express it in numbers, you know something about it." - Lord Kelvin (1824 - 1907)

Prince Louis - Victor de Broglie
(born Dieppe, France, 1892 - 1987)
French Academy, Permanent Secretary of the Academy of Sciences
Nobel Prize in Physics 1929 for mathematically identifying
the wave nature of matter at high velocity or wave-particle duality

§ A short excerpt from "What is matter?", by Erwin Schrödinger, 1952:

"We no longer contrast matter with forces and field of force as different entities, we know now that these concepts have to be merged. It is true, we call a spatial region free of matter, call it empty if there is nothing but a gravitational field. But space is never really empty because even far away in the universe there is starlight, and that is matter [...] "

source: Erwin Schrödinger, 1952 Audio - document from the lecture "Was ist Materie?" English translation: taken from the lecture "Our Conception of Matter" given in 1952 organized by Rencontres Internationales de Genève, Geneva, Switzerland

§ de Broglie's Quantum Equation:

de Broglie's original quantum equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\left(\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\right)v} = \frac{h}{m_o v}\sqrt{1-\frac{v^2}{c^2}}\ ,\ \text{quanta wave length utilizing relativistic mass}\,,$$

where

m_o is rest or proper mass
v is wave - particle velocity
p is quanta momentum
h is Planck's Constant $= 6.626 \times 10^{-34}$ Joule - sec

$$\Longrightarrow \frac{1}{\lambda} = \frac{f}{c} = \frac{m_o v}{h} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\ \text{where } \lambda = \frac{c}{f}$$

$$\Longrightarrow \therefore\ f = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{1}{h} \cdot vc = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} c^2 \cdot \frac{1}{h} = mc^2 \cdot \frac{1}{h} = \frac{E}{h}\ ,\ \text{where wave - particle } v \to c \text{ and } E \text{ is total energy}$$

$$\Longrightarrow \therefore\ \boxed{E = hf},\ \text{a classic quantum energy relationship}$$

$$\Longrightarrow p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c},\ \text{from de Broglie above and } \frac{1}{\lambda} = \frac{f}{c}$$

$$\Longrightarrow \therefore\ \boxed{p = \frac{E}{c}},\ \text{another classic quantum relationship which by special relativity is also derived as follows in two ways:}$$

(1) $m = \frac{E}{c^2} \implies p = mv = \frac{E}{c^2}v = \frac{E}{c}$, as $v \to c$

(2) $E^2 - c^2p^2 = (m_o c^2)^2 \implies E^2 = c^2p^2 + (0)^2$, since the photon's rest or proper mass m_o is zero as $v \to c$

$\implies p = \frac{E}{c}$

see: Some Quantum Consequences of $E = mc^2$

Therefore,

$$\boxed{E = hf = h \cdot \frac{c}{\lambda} \implies \lambda = \frac{hc}{E} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{mc}, \text{ as } v \to c \implies \therefore\ E = mc^2}$$

Examples of Determining de Broglie's Wavelength*

Problem 1: Find the de Broglie wavelength of a wave - particle whose total energy,

$$E = mc^2 = E_0 + T_{k.e.}, \text{ where}$$
$$E_0 = m_0c^2 = \text{"rest energy"}$$
$$T_{k.e.} = m_0c^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right], \text{ relativistic kinetic energy}, k.e.$$

is 3.5×10^{-19} Joule.

Solution:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\,\text{Joule - sec})(2.998 \times 10^{8}\,\text{m/sec})}{3.5 \times 10^{-19}\,\text{Joule}}$$
$$= \frac{1.9798 \times 10^{-25}\,\text{Joule - meter}}{3.5 \times 10^{-19}\,\text{Joule}}$$
$$\implies \therefore\ \lambda = 5.65657 \times 10^{-7}\text{meter wavelength}$$

Problem 2: Find the de Broglie wavelength of earth whose rest or proper mass is 5.9736×10^{24} kg.

Solution:

$$\lambda_{\text{earth}} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}v} = \frac{h}{m_o v}, \text{ since } \sqrt{1-\frac{v^2}{c^2}} \to 1 \text{ as } v << c$$
$$= \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{\underbrace{(5.9736 \times 10^{24}\,\text{kg})}_{\text{rest mass of earth}}\underbrace{(29.88578 \times 10^{3}\text{m/sec})}_{\text{earth's orbit velocity*}}}$$
$$= 3.7115 \times 10^{-63}\frac{\text{J}\cdot\text{sec}}{\text{kg}\cdot\text{m}\cdot\text{sec}^{-1}} \times \frac{\text{kg}\cdot\frac{\text{m}^2}{\text{sec}^2}}{\text{J}}, \text{ using the International System of Units for 1.0 Joule of energy}$$

$$\implies \therefore\ \lambda_{\text{earth}} = 3.7115 \times 10^{-63}\,\text{meter!!}$$

* $$\frac{\text{earth velocity}}{\text{speed of light}} = \frac{29.88578 \times 10^3\,\text{m/sec}}{299,792,458\,\text{m/sec}} = 0.0099688\% \approx 0$$

* note: some of these examples are used in the future upcoming Relativity Science Calculator Mac application for de Broglie's quantum equations.

§ References:

1. "Recherches Sur la Théorie des Quanta" (Research on the Theory of the Quanta), par M. Louis de Broglie, Annales de Physique, 10^e Série, Tomé **III**, Janvier - Février, 1925, pgs 21 - 128, the original French version of de Broglie's original 1927 Phd. thesis

2. An English translated version of de Broglie's original 1927 Phd. thesis, by A.F. Kracklauer, 2004